Robust Clipping for OFDM Transmissions over Memoryless Impulsive Noise Channels

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***Abstract*—The detriment arising from strong and frequently occurring impulses over an Orthogonal Frequency Division Multiplexing system is paramount because signals on sub-carriers appear to be corrupted simultaneously. To overcome this obstacle, clipping has been reported as an effective approach. Unlike previous works, this work derived the clipping threshold without assuming the a priori knowledge of the probability density function (PDF) of impulsive noise, which is usually unobtainable in precise measure in most practical scenarios, and may change rapidly over time. Then, a decoding metric accommodated to the clipping effect was derived to realize coding gain. To attest the proposed scheme, this study conducted computer simulations in compliance with the IEEE 1901 standard. For various impulse noise models under consideration, the proposed scheme was promisingly on par with its conventional counterpart, the clipping threshold of which, however, relies on an assumed PDF.**

***Index Terms*—Clipping threshold, impulsive noise, OFDM.**

I. Introduction

INVESTIGATION of data delivery under non-Gaussian noise in wireless communications, power line communications (PLC), and digital subscriber line loops has received much attention. In essence, nuisances such as human-made electromagnetic interference and atmospheric noise limit stable communication link quality. Orthogonal Frequency Division Multiplexing (OFDM) may be applied to reduce the adverse effect of impulsive interference caused by the spread of noise energy across sub-carriers induced by the Fast Fourier Transform (FFT) operation. Nevertheless, OFDM is outperformed by its single-carrier counterpart in uncoded symbol error probability within the context of excessive impulsive noise energy, coupled with a fairly high arrival probability of impulses [1], which likely exists in contemporary communication systems [2].

A simple but effective means to mitigate impulsive noise in OFDM systems is to equip the front-end receiver with a memoryless nonlinearity. A number of studies, including [3], [4] attempted to determine the optimal threshold of clipping by applying Bussgang's theorem, which produced considerable symbol error rate performance gain. A decision-directed impulsive noise cancelation scheme was proposed in [5] to address impulsive noise with a power level commensurate with the OFDM signal. Iterative interference cancelation for clipping noise arising from blanking nonlinearity was investigated in [6]. Other than clipping, channel coding [7], [8] is effective in tackling impulsive noise and approximating performance to the capacity limit. However, the coding gain may be invalidated under certain practical scenarios, wherein the impulsive noise has an excessive energy level and a fairly high arrival probability, and wherein the coding epoch is on a per-OFDM symbol basis [9]. A limiter, the clipping threshold of which was devised based on detection theory [9], [10], can enhance bit error rate (BER) performance significantly when placed in front of the FFT demodulator and turbo decoder.

These schemes for measuring the clipping threshold assumed perfect knowledge of the probability density function (PDF) of the impulsive noise model, which is not only too difficult and costly to obtain precisely, but may in reality also change over time [2]. The present work attempts to overcome this drawback by relaxing the assumption of having perfect statistical knowledge of impulsive noise at the receiver. Inspired by an efficient decoding scheme in [11], which approximates BER to the maximum-likelihood decoding method, regardless of which impulsive noise model the transmitted signal is subject to, the clipping threshold was measured with only a rough range of the arrival probability of impulses available at the receiver. Furthermore, an adapted log-likelihood ratio (LLR) metric for turbo decoding was derived, considering the clipping effect on both OFDM signaling and impulse prior to turbo decoding. Simulations conducted in compliance with the IEEE 1901 [12] PLC standard show that the proposed hybrid approach of using clipping and channel coding is not only robust, but can also induce BER commensurate with that of [9], which relies on the assumed PDF of impulsive noise.

The rest of this paper is organized as follows: Section II introduces a review of common impulsive noise models, and then presents the system framework; Section III details the schemes for deriving the clipping threshold, as well as an adopted decoding metric; Section IV provides the simulation results; and lastly, Section V offers a conclusion.

II. System Model

A. Prevalent Impulsive Noise Channels

We consider digital signaling transmitted over a memoryless impulsive noise channel, where the noise sample z_k is the

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combination of additive white Gaussian noise (AWGN) g_k and impulsive noise i_k such as $z_k = g_k + i_k$, where k is the time index. In mathematical form, the sample of the Bernoulli-Gaussian (B-G) model [1] is represented by $z_k = g_k + b_k\omega_k$, where $b_k \in \{0, 1\}$ is a Bernoulli random variable with $\Pr(b_k = 1) = p_b$ the probability of impulse occurrences and ω_k is an independent and identically distributed (*i.i.d.*) Gaussian random process with a mean zero and variance $\frac{N_0}{2}\Gamma$, where N_0 is the single-sided power spectral density of AWGN, and Γ stands for the mean power ratio between the impulsive noise component and AWGN component for the B-G model. The PDF of this Gaussian mixture model is thus denoted by $\Pr_{z_k}(x) = (1 - p_b)\mathcal{N}(x; 0, \frac{N_0}{2}) + p_b\mathcal{N}(x; 0, \frac{N_0}{2}(1 + \Gamma))$, where $\mathcal{N}(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ is the Gaussian PDF of a random variable x with mean μ and variance σ^2.

In addition, popularly adopted in the literature for characterizing impulsive noises is the Middleton Class-A (M-CA) noise model [13], which describes the PDF of *i.i.d.* noise samples z_k as $\Pr_{z_k}(x) = \sum_{m=0}^{\infty} \alpha_m \mathcal{N}(x; 0, \sigma_m^2)$, where $\alpha_m = e^{-A}\frac{A^m}{m!}$ and $\sigma_m^2 = \frac{N_0}{2}(1 + \frac{m}{\Lambda A})$, where A is the impulsive index, and Λ is the mean power ratio between the AWGN component and the impulsive noise component for the M-CA model [2].

B. System Framework & Problem Statement

We used a double binary turbo convolutional encoder, followed by a channel interleaver at the front end of the transmitter. Subsequently, the coded bit stream was partitioned into segments in accordance with the Quadrature Phase Shift Keying (QPSK) modulation with Gray mapping. After collecting M consecutive modulated symbols X_m, where m is the sub-carrier index for the Inverse Fast Fourier Transform (IFFT) implementation, the output of IFFT can be expressed as $x_k = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} X_m e^{\frac{j2\pi mk}{M}}, \ (0 \le k \le M - 1)$.

Assuming perfect sampling time and frequency control, the OFDM symbol, subject to memoryless impulsive noise sequence n_k, can be denoted by $r_k = x_k + n_k, \ k = 0, 1, \ldots, M - 1$, after front-end filtering, where both real and imaginary parts of n_k, i.e., $\Re(n_k)$ and $\Im(n_k)$, have the same distribution and can be modeled in the same manner as the real-value noise z_k, presented in Section II-A. After conducting FFT over M successive samples r_k, one can arrive at $R_m = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} r_k e^{\frac{-j2\pi mk}{M}} = X_m + N_m$, where $N_m = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} n_k e^{\frac{-j2\pi mk}{M}}$. Under the B-G noise model, the PDF of $N_{m_R} = \Re(N_m)$ $(0 \le m \le M - 1)$, which is no longer AWGN, can be written as [1]

$$\Pr{}_{N_{m_R}}(x) = \sum_{k=0}^{M} \binom{M}{k} (p_b)^k (1 - p_b)^{M-k} \mathcal{N}(x; 0, \sigma_k^2) \ , \qquad (1)$$

where $\sigma_k^2 = \frac{N_0}{2}\left(1 + \frac{k\Gamma}{M}\right)$ and $\binom{M}{k} = \frac{M!}{(M-k)!k!}$. In this perspective, one can readily obtain the variance of noise sample N_{m_R}, on average, as

$$\sigma_{N_{m_R}}^2 = \sum_{k=0}^{M} \binom{M}{k} (p_b)^k (1 - p_b)^{M-k} \sigma_k^2 = \frac{N_0}{2}(1 + p_b\Gamma) \ . \qquad (2)$$

For a sufficiently large value of M, one can invoke the Central Limit Theorem (C.L.T.) and construe the LLR channel measurement element associated with the output LLR of the turbo decoder as

$$L_c(R_{m_R}) = \ln\left(\frac{\Pr(R_{m_R} | X_{m_R} = \sqrt{E_s/2})}{\Pr(R_{m_R} | X_{m_R} = -\sqrt{E_s/2})}\right) = \sqrt{2E_s}\frac{R_{m_R}}{\sigma_{N_{m_R}}^2} \ , \qquad (3)$$

where E_s is the modulated symbol energy. Assuming perfect synchronization, similar formats on the PDF (1) and output LLR (3) can be derived for $\Im(N_m)$, which, however, plagues $\Im(X_m)$.

III. Efficient Clipping Against Impulses

Under the B-G model with large p_b and Γ, one can observe from (3) that each modulated symbol X_m $(0 \le m \le M - 1)$ is likely to be corrupted, irrespective of the sub-carrier, due to the enlarged $\sigma_{N_{m_R}}^2$ in (2) as stressed in [1]. This drawback further invalidates the channel coding gain in the proposed framework, where a codeword only encompasses an OFDM symbol, prompting an adoption of a limiter prior to the FFT operator to suppress impulses. Arguments on the effectiveness of the turbo decoder can be applied to OFDM transmission under the M-CA noise model with a large A as well as strong impulses, which in turn induces a small Λ.

To suppress impulses, we followed [3], who used a memoryless limiter with real and imaginary clipping

$$\overline{r}_{k_I} = \begin{cases} r_{k_I} & |r_{k_I}| < \gamma \\ \gamma \cos(\angle(r_{k_I})) & \text{otherwise} \end{cases} \ , \qquad (4)$$

where γ is the clipping threshold, and $\angle(r_{k_I})$ is the phase (in units of radian) of the I-phase component at the k-th output of IFFT, r_{k_I}. Nevertheless, unlike their methods, we forwent using the PDF of the impulsive noise, and instead relied only on the arrival probability of impulses to devise threshold γ, such that the variance of the clipped signals after FFT is properly reduced, irrespective of the impulsive noise models.

To illustrate, the B-G noise model was assumed in order to highlight the concept of designing γ. As a result of the assumed Gaussianity of x_{k_I} with zero mean and variance $E_s/2$, the received signal r_{k_I} is zero-mean Gaussian with variance σ_c^2 with probability $1 - p_b$ and $\sigma_c^2 + \frac{N_0}{2}\Gamma$ with probability p_b, where $\sigma_c^2 = (N_0 + E_s)/2$. The decision statistic on the basis of the Bayes criterion can be expressed as

$$\Lambda_c(r_{k_I}) = \frac{\Pr(H_1 | r_{k_I})}{\Pr(H_0 | r_{k_I})} \ , \qquad (5)$$

where H_0 is associated with the absence of impulses in the received sample, while the presence of impulse is with regards to H_1. It is clear the a posteriori probability $\Pr(H_0 | r_{k_I}) = \frac{1-p}{\Pr(r_{k_I})}\mathcal{N}(r_{k_I}; 0, \sigma_c^2)$.[1] However, as opposed to [14], the insufficient information of statistics on impulsive noise leads to the expression of the a posteriori probability $\Pr(H_1 | r_{k_I})$ by $p / \Pr(r_{k_I})$. Consequently, one can readily obtain the decision statistic (5) as

$$\Lambda_c(r_{k_I}) = \frac{p}{(1 - p)\mathcal{N}(r_{k_I}; 0, \sigma_c^2)} \begin{array}{ll} \ge 1 & \text{choose } H_1 \\ < 1 & \text{choose } H_0 \end{array} \ . \qquad (6)$$

[1]Indeed, p is approximately equal to p_b under the B-G model and to the impulsive index A under the M-CA model.

From (6), the clipping threshold γ can be deduced such that $\Lambda_c(\gamma) = 1$ is satisfied, which indicates that γ is measured by

$$(1-p)\mathcal{N}(\gamma; 0, \sigma_c^2) = p \ . \tag{7}$$

Note that γ is independent of the model parameter Γ or Λ.

The clipped sample can be expressed either by $\bar{r}_{k_I} = x_{k_I} + g_{k_I}$ at a probability of $1-p$, where g_{k_I} is the AWGN, or by $\bar{r}_{k_I} = x_{k_I} + \gamma$ or $\bar{r}_{k_I} = x_{k_I} - \gamma$ at $p/2$ probability of each. In view of this, one can construe that signaling x_{k_I} within the IFFT-domain sequence r_{k_I} is likely to remain intact through a limiter (4) with a proper clipping threshold (7), prompting us to examine the statistical behavior of the remaining interference induced by the limiter to x_{k_I}. The mean value of $\bar{r}_{k_I}$ conditioned on x_{k_I}, i.e. $E[\bar{r}_{k_I}|x_{k_I}]$, is computed by $(1-p)E[g_{k_I}] + \frac{p}{2}(-\gamma) + \frac{p}{2}\gamma = 0$, and the conditional variance is measured straightforwardly by

$$E[\bar{r}_{k_I}^2|x_{k_I}] = (1-p)E[g_{k_I}^2] + \frac{p}{2}(-\gamma)^2 + \frac{p}{2}\gamma^2 = (1-p)\frac{N_0}{2} + p\gamma^2. \tag{8}$$

The derivation of the clipping threshold for the Q-component of r_k, i.e., r_{k_Q}, as well as the ensuing arguments on the interference with it can also be made, but they are omitted here because of space limitations.

To re-assess the LLR value of the turbo decoder resulting from a limiter placed at the front-end receiver, one can first investigate the FFT of clipped sequence $\bar{r}_k$, which can be obtained by performing $\bar{R}_m = \frac{1}{\sqrt{M}}\sum_{k=0}^{M-1} \bar{r}_k e^{-\frac{j2\pi km}{M}}$ to arrive at $\bar{R}_m = X_m + N_m^{(\gamma)}$, where we refer $N_m^{(\gamma)}$ to the previous paragraph as the residual noise with respect to interference after the clipping operation. By again invoking the C.L.T. because of a large M, the noise term after FFT implementation, i.e., $N_m^{(\gamma)}$, can be modeled by the Gaussian random process. With perfect time and frequency control, the variance of real and imaginary parts of $N_m^{(\gamma)}$, denoted by σ_ε^2, is equal to $(1-p)\frac{N_0}{2} + p\gamma^2$ from (8). Consequently, the PDF of $\bar{R}_{m_R} = \Re(\bar{R}_m)$ conditioned on $X_{m_R} = \pm\sqrt{E_s/2}$ is simply Gaussian and can be expressed as $\Pr_{\bar{R}_{m_R}}(x|X_{m_R} = \pm\sqrt{E_s/2}) = \mathcal{N}(x; \pm\sqrt{E_s/2}, (1-p)\frac{N_0}{2} + p\gamma^2)$. The LLR channel measurement element associated with the output LLR of the turbo decoder for the induced clipped signal $\bar{R}_{m_R}$ can be obtained straightforwardly by

$$L_c(\bar{R}_{m_R}) = \sqrt{2E_s}\frac{\overline{R}_{m_R}}{(1-p)\frac{N_0}{2} + p\gamma^2} \ . \tag{9}$$

As opposed to [9], the denominator of (9) does not account for modulated symbol energy E_s, and is not dictated by the parameter of any impulsive noise model other than p, which, as revealed through simulation in Section IV, results in negligible performance loss when deviating significantly from the true probability such as p_b of the B-G noise model, despite the proposed receiver being unaware of which impulsive noise model the transmitted data are faced with.

IV. Simulation Results

In compliance with the Frame Control field specified in the IEEE 1901 PLC standard [12], we used a double binary turbo convolutional encoder with no puncturing assumed such that the code rate is rendered at $\frac{1}{2}$. The number of information bits

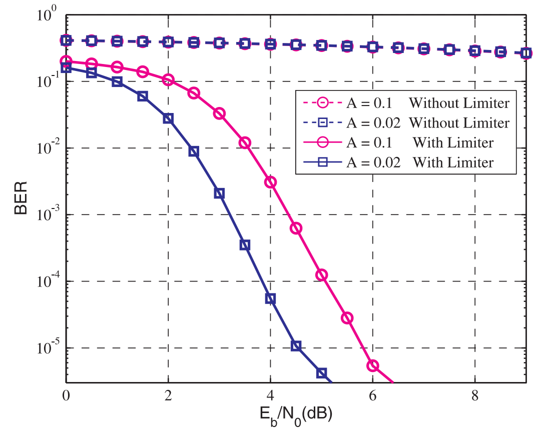

Fig. 1. BER versus E_b/N_0 (dB) for OFDM transmissions over the M-CA noise channel when $\Lambda = 0.05$.

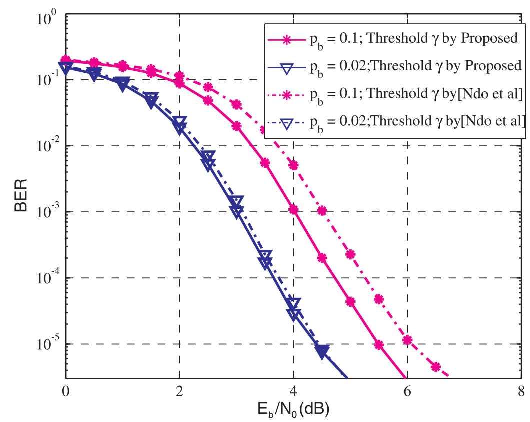

Fig. 2. BER performance comparison with the existing method [9] for OFDM transmissions over the B-G noise channel when $\Gamma = 20$.

in a frame is set to 128. Under memoryless impulsive noise channel models such as the M-CA and B-G noise models[2] mentioned in Section II-A, simulations were performed and related results, for BER versus E_b/N_0 (dB), where E_b is the information bit energy, were exhibited to justify the efficacy of the proposed clipping scheme. Furthermore, each BER curve plotted in the following figures was induced by applying the turbo decoder with nine iterations.

Performance gain by means of the limiter with the proposed threshold is shown in Fig. 1, where Λ is 0.05, but the impulsive index A is varied. Whereas one can observe that dash lines, associated with no-limiter receiver under the M-CA noise model, are fairly flat over the plotted E_b/N_0 range, the BER curve induced by using the limiter, in relation to the solid line with "$\circ$" for larger $A = 0.1$, declines to 10^{-5} at 5.5dB E_b/N_0, indicating that the proposed threshold γ is effective for the limiter even in the absence of the PDF of impulsive noise. As expected, a lower A, associated with an occurrent rate of impulse, can lead to enhanced BER performance, as seen in the solid line with "$\square$".

The comparison of BER performance with an existing clipping approach [9], which is derived from detection theory but uses the impulsive noise model, was conducted under the B-G model with $\Gamma = 20$ but p_b varied, and the result is plotted

[2]Recall that, in both models, the background noise is the AWGN.

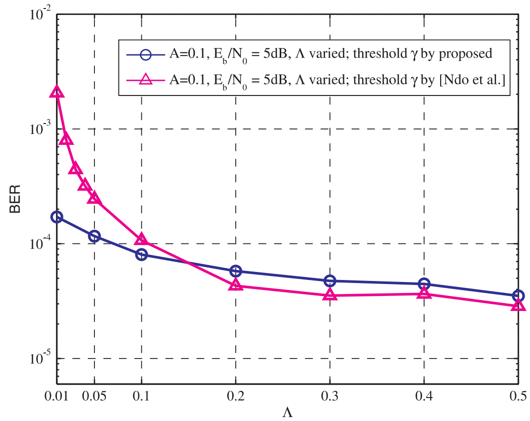

Fig. 3. BER versus Λ, the mean power ratio between the AWGN and impulsive noise, for OFDM transmissions over the M-CA noise channel.

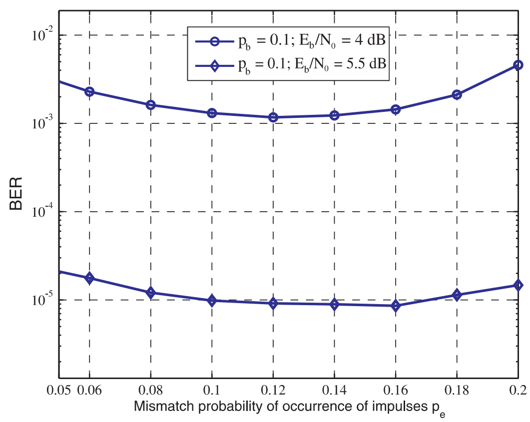

Fig. 4. BER versus p_e, the mismatch probability of occurrence of impulses assumed at receiver, for OFDM transmissions over the B-G noise channel when $p_b = 0.1$ and $\Gamma = 20$.

in Fig. 2. At the higher $p_b = 0.1$, the proposed scheme, evident in the solid line with "$*$", outperformed that of [9], associated with the dash-dot line with "$*$", by 0.5dB E_b/N_0 at 10^{-5} BER, whereas both schemes performed equally at $p_b = 0.02$, corresponding to all lines with "$\bigtriangledown$".

Fig. 3 shows the BER result compared to [9] at 5dB E_b/N_0 over the M-CA model with $A = 0.1$ as a function of Λ. As the impulse subdues; i.e., as Λ is increased, both BER curves declined and became fairly close to each other. Nevertheless, with stronger impulse, the induced BER curve by the proposed scheme, in relation to the line with "$\circ$", does not fluctuate as much as that of [9], associated with the line with "$\triangle$", which, however, exploits the M-CA model. Note that the PDF of the received signal r_{k_I}, computed in [9] for deriving the threshold γ, deviates excessively from its precise one in the regime of large A (or p_b) and strong impulse (small Γ), which reflects a worse BER performance result, as shown in Fig. 2 (p_b = 0.1) and Fig. 3 ($\Lambda < 0.15$). On the contrary, our clipping method is more robust for not being engaged in the PDF measurement to render γ.

Despite assuming the probability of impulse occurrences p_b in the B-G model or impulsive index A in the M-CA model in deriving the clipping threshold (7) in Section II, the proposed simulation allowed for probability mismatch (i.e., the probability of occurrence of impulses assumed at receiver, namely p in (7) is not identical to the true probability p_b of the B-G model in generating impulsive noise), to justify the robustness of the proposed scheme against strong impulse. Fig. 4 shows that, despite the receiver being unaware of the B-G noise model the transmitted data sequence is subjected to, the BER curves are still on the same order when the E_b/N_0 values are set to 4 and 5.5 dBs, respectively, where the mismatch probability p ranges from up to two times to only half the probability $p_b = 0.1$ (i.e., p varies from 0.05 to 0.2).

V. Conclusion

Using only an approximate knowledge of the arrival probability of impulses to render the clipping threshold, this study proposed a robust clipping scheme for OFDM transmissions over memoryless impulsive noises, characterized by frequent occurrence and relatively strong power to that of background noise. Through computer simulations over multiple impulsive noise models, the findings show that the proposed robust clipping scheme, when coupled with a turbo decoder using adapted log-likelihood ratio metrics, is on par with and even outperform its conventional counterpart, which assumes full statistical knowledge of impulsive noise.

References

[1] M. Ghosh, "Analysis of the effect of impulse noise on multicarrier and single carrier QAM systems," *IEEE Trans. Commun.*, pp. 145–147, Feb 1996.

[2] H. C. Ferreira, L. Lampe, J. Newbury, and T. G. Swart, *Power Line Communications: Theory and Applications for Narrowband and Broadband Communications over Power Lines*. John Wiley and Sons, 2010.

[3] H. A. Suraweera, C. Chai, J. Shentu, and J. Armstrong, "Analysis of impulse noise mitigation techniques for digital television systems," in *Proc. 2003 Int. OFDM Workshop*, pp. 172–176.

[4] S. V. Zhidkov, "Analysis and comparison of several simple impulsive noise mitigation schemes for OFDM receivers," *IEEE Trans. Commun.*, pp. 5–9, Jan. 2008.

[5] J. Armstrong and H. A. Suraweera, "Impulse noise mitigation for OFDM using decision directed noise estimation," in *Proc. 2004 IEEE Int. Symp. Spread Spectrum Techniques and Applications*, pp. 174–178.

[6] C.-H. Yih, "Iterative interference cancellation for OFDM signals with blanking nonlinearity in impulsive noise channels," *IEEE Signal Process. Lett.*, pp. 147–150, Mar. 2012.

[7] J. Haring and A. J. H. Vinck, "Coding for impulsive noise channels," in *Proc. 2001 ISPLC*, pp. 103–108.

[8] D. Umehara, H. Yamaguchi, and Y. Morihiro, "Turbo decoding in impulsive noise environment," in *Proc. IEEE 2004 Global Telecom. Conf.*, pp. 194–198.

[9] G. Ndo, P. Siohan, M.-H. Hamon, and J. Horard, "Optimization of turbo decoding performance in the presence of impulsive noise using soft limitation at the receiver side," in *Proc. 2008 IEEE Global Telecom. Conf.*, pp. 1–5.

[10] G. Ndo, P. Siohan, and M.-H. Hamon, "Adaptive noise mitigation in impulsive environment: application to power-line communications," *IEEE Trans. Power Delivery*, pp. 647–656, Apr. 2010.

[11] D.-F. Tseng, Y. S. Han, W. H. Mow, and J. Deng, "Efficient decoding over power-line channels," in *Proc. 2011 Int'l Workshop on Signal Design and its Applications in Communications*, pp. 138–141.

[12] IEEE Std 1901-2010, IEEE Standard for Broadband over Power Line Networks: Medium Access Control and Physical Layer Specifications, Dec. 2010.

[13] D. Middleton, "Canonical and quasi-canonical probability models of class A interference," *IEEE Trans. Electromagn. Compat.*, pp. 76–106, May 1983.

[14] W. Henkel and F. Hu, "OFDM and analog RS/BCH codes." Available: http://trsys.faculty.jacobs-university.de/english/hfn/OFDM05_henkel_hu.pdf.